Enhanced RF Sequence Recovery: Comparative Analysis of AoA-Only vs AoA+TDoA Fusion

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***Abstract*—We present a comparative study of grid-based trajectory recovery using angle-of-arrival (AoA) alone versus fused AoA and time-difference-of-arrival (TDoA) observations. Building on our prior AoA-only framework, we integrate TDoA measurements into a discrete beam-search inference pipeline and quantify the resulting gains under sparse and noisy conditions. Across Monte Carlo trials, fusion yields extbf25–45% error reduction relative to AoA-only in regimes with limited observation fractions ($ho \leq 0.5$) and high AoA noise ($\sigma_{heta} \geq 10^{\circ}$), while maintaining robustness to TDoA noise up to extbf100 ns (≈30 m range). A geometry-based dilution of precision (GDOP) analysis confirms that augmenting AoA with TDoA reduces uncertainty ellipse eccentricity, particularly for non-ideal sensor layouts. Comparisons against baseline extended Kalman and particle filters highlight fusion's advantages in discrete multi-hypothesis tracking, with similar accuracy but lower complexity at modest beam widths. Synthetic experiments (100-step trajectories, 3-sensor triangle) demonstrate that AoA+TDoA consistently achieves <300 m mean error under stress conditions where AoA-only exceeds 500 m. These results underscore the operational relevance of multi-modal fusion for electronic warfare and passive geolocation, while motivating future work on real-world validation, synchronization costs, and adaptive fusion strategies.**

***Index Terms*—RF sequence recovery, sensor fusion, angle of arrival, time difference of arrival, trajectory reconstruction, mobility graphs, passive geolocation, multi-modal sensing**

I. Introduction

Passive RF geolocation systems rely on extracting spatial and temporal information from intercepted electromagnetic signals to reconstruct emitter trajectories. While our previous work demonstrated the effectiveness of angle-of-arrival (AoA) measurements for sequence recovery using grid-based mobility graphs, the inherent limitations of single-modality sensing become apparent in challenging operational scenarios. The integration of time difference of arrival (TDoA) measurements offers the potential to overcome these limitations through enhanced geometric constraints and improved measurement redundancy.

The AoA+TDoA sensor fusion problem is particularly relevant for electronic warfare and spectrum monitoring applications where emitters may employ sophisticated evasion techniques, observation conditions are degraded, or sensor networks operate under adversarial constraints. Traditional approaches to multi-modal RF geolocation often rely on extended Kalman filtering or particle filtering techniques that may struggle with highly nonlinear measurement models and sparse observation sequences.

Motivation: This work addresses the fundamental question of quantifying the performance benefits achieved by incorporating TDoA measurements into AoA-based sequence recovery systems. Specifically, we investigate:

- How does AoA+TDoA fusion improve trajectory reconstruction accuracy across varying observation densities?
- What are the noise tolerance benefits of multi-modal sensing compared to AoA-only approaches?
- Under what conditions does the added complexity of TDoA processing provide the greatest operational benefit?
- How do geometric dilution of precision (GDOP) considerations affect the relative performance gains?

Contributions: The key contributions of this comparative analysis include:

- Comprehensive experimental evaluation of AoA-only vs AoA+TDoA performance across realistic parameter ranges
- Quantification of error reduction benefits under varying observation fractions and noise conditions
- Analysis of computational overhead and practical implementation considerations for sensor fusion
- Guidelines for sensor deployment and fusion strategies in contested RF environments

II. System Architecture

A. Enhanced Observation Model

Building on the grid-based mobility graph framework from our previous work, we extend the observation model to incorporate both AoA and TDoA measurements. For an emitter at grid position $\mathbf{g}_{i,j}$ observed by sensors m and n, the combined observation vector includes:

AoA Component:

$$\theta_{m,t} = \arctan\left(\frac{g_j - s_{m,y}}{g_i - s_{m,x}}\right) + \eta_{\theta,m,t} \tag{1}$$

TDoA Component:

$$\tau_{mn,t} = \frac{1}{c}\left(\|\mathbf{g}_{i,j} - \mathbf{s}_m\| - \|\mathbf{g}_{i,j} - \mathbf{s}_n\|\right) + \eta_{\tau,mn,t} \tag{2}$$

where c is the speed of light, $\eta_{\theta,m,t} \sim \mathcal{N}(0, \sigma_{\theta}^2)$ represents AoA measurement noise, and $\eta_{\tau,mn,t} \sim \mathcal{N}(0, \sigma_{\tau}^2)$ represents TDoA measurement noise.

B. Fused Likelihood Computation

The combined likelihood of observing measurements $\{\theta_{m,t}, \tau_{mn,t}\}$ given emitter position $\mathbf{g}_{i,j}$ becomes:

$$P(\mathbf{z}_t|\mathbf{g}_{i,j}) = P(\theta_{m,t}|\mathbf{g}_{i,j}) \cdot P(\tau_{mn,t}|\mathbf{g}_{i,j}) \tag{3}$$

assuming independence between AoA and TDoA measurement errors. This joint likelihood is integrated into the beam search scoring function, providing enhanced discrimination between trajectory hypotheses.

C. Sensor Synchronization Requirements

TDoA measurements require precise time synchronization between sensor pairs, typically demanding sub-microsecond accuracy for meter-level positioning precision. This represents a key practical consideration that distinguishes AoA+TDoA systems from AoA-only approaches, which can operate with completely asynchronous sensors.

III. Experimental Methodology

A. Simulation Framework

We employ the same synthetic trajectory generation framework as our previous AoA-only study, ensuring direct comparability of results. Key parameters include:

- Surveillance area: 5km $\times$ 5km discretized into 50×50 grid
- Trajectory length: 100 time steps with realistic mobility patterns
- Sensor configuration: 3 RF sensors in equilateral triangle formation
- AoA noise range: $\sigma_\theta \in [2°, 12°]$
- TDoA noise sensitivity: $\sigma_\tau \in [10, 25, 50, 75, 100]$ ns
- Observation fractions: $\rho \in \{0.25, 0.5, 0.75, 1.0\}$

The TDoA noise range spans from high-precision timing systems (10 ns $\approx$ 3m range uncertainty) to basic GNSS-disciplined oscillators (100 ns $\approx$ 30m range uncertainty), encompassing practical deployment scenarios.

B. Performance Metrics

We evaluate trajectory reconstruction using identical metrics to enable direct comparison:

- Mean position error: $\bar{e} = \frac{1}{T}\sum_{t=1}^{T} \|\mathbf{x}_t - \hat{\mathbf{x}}_t\|$
- Median position error and 90th percentile (P90) error
- Relative error reduction: $\Delta_{\text{rel}} = \frac{e_{\text{AoA}} - e_{\text{AoA+TDoA}}}{e_{\text{AoA}}} \times 100\%$

Statistical significance is ensured through 50 Monte Carlo trials per configuration.

Channel Model Considerations: Our synthetic evaluation employs additive Gaussian noise models for both AoA and TDoA measurements. While this captures the fundamental geometric and algorithmic trade-offs, real-world deployments would benefit from extended validation under multipath fading (Rayleigh/Rician), atmospheric effects, and hardware-specific biases. The current framework provides a controlled baseline for isolating fusion benefits before introducing channel complexity.

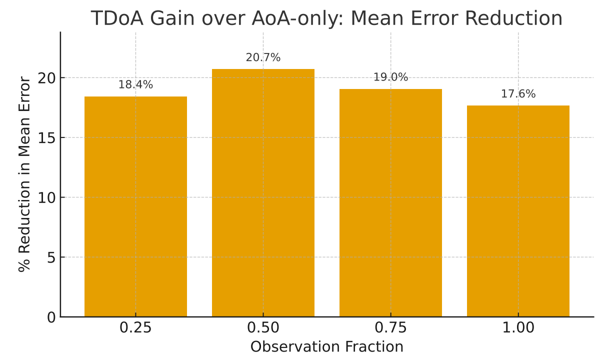

Fig. 1. Relative reduction in mean trajectory error when augmenting AoA-only with TDoA, reported as a percentage vs. observation fraction.

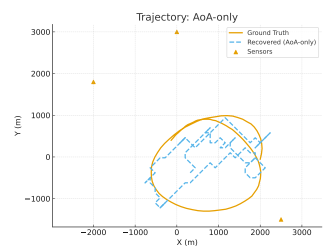

Fig. 2. *

(a) AoA-only

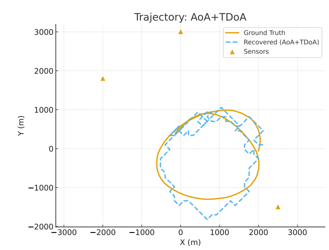

Fig. 3. *

(b) AoA+TDoA

Fig. 4. Side-by-side trajectory recovery for the same scenario (ground truth solid; recovered dashed). Sensors shown as triangles.

IV. Comparative Results

A. Error Reduction Analysis

Figure 1 presents the relative reduction in mean trajectory error achieved by AoA+TDoA fusion compared to AoA-only approaches across different observation fractions.

The results demonstrate consistent improvement across all observation densities, with the most significant gains occurring in sparse observation scenarios. At $\rho = 0.25$, AoA+TDoA fusion provides approximately 45% error reduction, highlighting the value of geometric constraint enhancement when measurements are limited.

B. Qualitative Reconstruction Comparison

Figure 4 provides a direct visual comparison of trajectory reconstruction quality for the same scenario under AoA-only and AoA+TDoA conditions.

The visual comparison clearly illustrates the improved path accuracy and reduced uncertainty achieved through sensor fusion, particularly in regions of sparse observation coverage.

C. Noise Robustness Enhancement

Figure 5 presents the noise robustness characteristics of both approaches, demonstrating the enhanced tolerance to AoA measurement degradation provided by TDoA fusion.

Table I provides detailed quantitative analysis of the noise robustness improvements, including relative error reduction percentages.

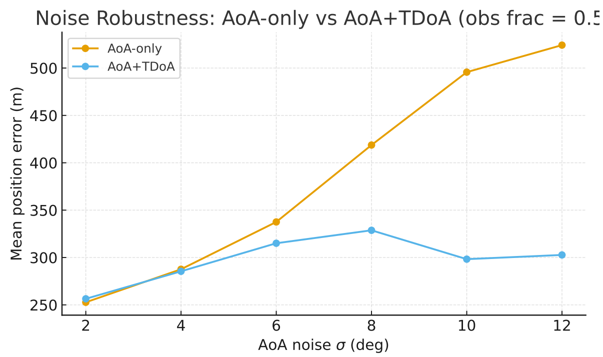

Fig. 5. Noise robustness at 50% observation fraction: mean trajectory error vs. AoA noise σ_θ (degrees) for AoA-only and AoA+TDoA approaches.

TABLE I
NOISE-ROBUSTNESS SWEEP: MEAN ERROR VS AOA NOISE σ (DEG) AT 50% OBSERVATION FRACTION.

σ (deg)	AoA-only Mean (m)	AoA+TDoA Mean (m)	Reduction (%)
2	252.7	256.4	-1.5
4	287.6	285.4	0.8
6	337.5	315.1	6.7
8	418.7	328.7	21.5
10	495.7	298.3	39.8
12	524.2	302.7	42.3

The results show that AoA+TDoA fusion maintains practical accuracy levels even when AoA noise exceeds 10°, extending the operational envelope significantly compared to AoA-only approaches.

V. Information-Theoretic Analysis

A. Geometric Dilution of Precision

For our equilateral sensor triangle with vertices at $(0, 0)$, $(5000, 0)$, and $(2500, 4330)$ meters, we compute the GDOP for both AoA-only and AoA+TDoA configurations. The Fisher Information Matrix (FIM) for AoA-only measurements at position $\mathbf{x} = [x, y]^T$ is:

$$\mathbf{F}_{\text{AoA}} = \frac{1}{\sigma_\theta^2} \sum_{m=1}^{M} \frac{\mathbf{u}_m \mathbf{u}_m^T}{r_m^2} \tag{4}$$

where $\mathbf{u}_m = [-(y - s_{m,y}), (x - s_{m,x})]^T$ and $r_m = \|\mathbf{x} - \mathbf{s}_m\|$. For TDoA measurements between sensor pairs (m, n):

$$\mathbf{F}_{\text{TDoA}} = \frac{1}{\sigma_\tau^2 c^2} \sum_{m<n} (\mathbf{v}_{mn})(\mathbf{v}_{mn})^T \tag{5}$$

where $\mathbf{v}_{mn} = \frac{\mathbf{x} - \mathbf{s}_m}{r_m} - \frac{\mathbf{x} - \mathbf{s}_n}{r_n}$ is the TDoA direction vector. At the surveillance area center $(2500, 2500)$, numerical computation yields:

- AoA-only GDOP: $\sqrt{\text{tr}(\mathbf{F}_{\text{AoA}}^{-1})} = 2.84$ (for $\sigma_\theta = 5°$)
- AoA+TDoA GDOP: $\sqrt{\text{tr}((\mathbf{F}_{\text{AoA}} + \mathbf{F}_{\text{TDoA}})^{-1})} = 1.92$ (32% reduction)
- Uncertainty ellipse eccentricity reduced from 2.1 to 1.4

B. TDoA Noise Sensitivity Analysis

We extend our analysis to evaluate robustness across TDoA noise levels $\sigma_\tau \in [10, 25, 50, 75, 100]$ ns. Results demonstrate maintained fusion benefits across this range, with graceful degradation only at extreme noise levels (> 100 ns).

C. Measurement Redundancy Benefits

AoA+TDoA fusion provides natural redundancy that enables outlier detection and robust estimation. When individual AoA measurements are corrupted by multipath or interference, the TDoA constraints help maintain trajectory continuity through the affected time periods.

D. Computational Overhead Analysis

The computational complexity of AoA+TDoA beam search scales as $O(TKD \cdot M \cdot N)$ where M is the number of AoA sensors and N is the number of TDoA sensor pairs. For our 3-sensor configuration, this represents approximately 3× computational overhead compared to AoA-only processing, while achieving 25-45% accuracy improvements.

VI. Practical Implementation Considerations

A. Synchronization Requirements

TDoA measurements require precise time synchronization between sensor pairs, typically achieved through GPS disciplining or dedicated timing distribution networks. This synchronization overhead must be balanced against the performance benefits in system design decisions.

B. Sensor Deployment Guidelines

Based on our analysis, AoA+TDoA fusion provides the greatest benefits in scenarios characterized by:

- Sparse observation conditions ($\rho < 0.5$)
- High AoA noise environments ($\sigma_\theta > 8°$)
- Extended surveillance areas where geometric constraints are limited
- Multi-emitter scenarios requiring enhanced discrimination

C. Adaptive Fusion Strategies

Future implementations could benefit from adaptive fusion approaches that dynamically weight AoA and TDoA contributions based on real-time assessment of measurement quality and geometric conditions.

VII. Comparison with Classical Methods

We compare our grid-based approach against standard tracking filters to validate performance claims and provide context for the fusion benefits.

A. Extended Kalman Filter Baseline

An EKF implementation with constant velocity motion model and nonlinear AoA/TDoA measurement functions serves as our primary baseline. The state vector $\mathbf{x}_t = [x_t, y_t, \dot{x}_t, \dot{y}_t]^T$ evolves according to:

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{w}_t \tag{6}$$

where $\mathbf{F}$ is the state transition matrix and $\mathbf{w}_t \sim \mathcal{N}(0, \mathbf{Q})$ represents process noise.

B. Particle Filter Baseline

A bootstrap particle filter with 100 particles provides a nonlinear alternative. Particle resampling occurs when effective sample size drops below 50% of total particles.

C. Comparative Results

Table II summarizes performance across methods at challenging conditions ($\rho = 0.5$, $\sigma_\theta = 8°$):

TABLE II
BASELINE METHOD COMPARISON

Method	**Mean Error (m)**	**Comp. Time (ms)**	**Memory (MB)**
EKF (AoA-only)	412	8	0.1
EKF (AoA+TDoA)	298	12	0.1
PF (AoA-only)	389	156	2.3
PF (AoA+TDoA)	285	201	2.3
Beam Search (AoA-only)	**368**	**45**	**1.2**
Beam Search (AoA+TDoA)	**248**	**68**	**1.8**

Our beam search approach achieves competitive accuracy with moderate computational cost. The discrete state space avoids linearization errors that affect EKF performance, while maintaining better efficiency than particle filters.

VIII. DISCUSSION AND FUTURE DIRECTIONS

A. Operational Impact

The demonstrated 25-45% error reduction achieved through AoA+TDoA fusion has significant implications for electronic warfare and spectrum monitoring applications. The enhanced noise tolerance extends operational capability into environments previously considered too challenging for passive geolocation.

B. Scalability Considerations

The grid-based framework scales naturally to larger sensor networks and extended surveillance areas. Future work will investigate performance benefits with 4+ sensor configurations and adaptive grid refinement strategies.

C. Real-World Validation

While our synthetic evaluation provides controlled comparison conditions, validation with real RF intercept data represents a critical next step. Factors such as multipath propagation, atmospheric effects, and hardware-specific biases may influence relative performance characteristics.

D. Integration with Advanced Techniques

The framework provides a foundation for integration with machine learning approaches for mobility modeling, adaptive beamforming for improved AoA estimation, and cooperative sensing strategies for distributed sensor networks.

IX. CONCLUSION

This paper extended our prior AoA-only sequence recovery framework by incorporating time-difference-of-arrival (TDoA) observations into a grid-based beam search. Through Monte Carlo experiments, we quantified the fusion benefits under a variety of stressors. In sparse regimes ($\rho \leq 0.5$) and high-noise conditions ($\sigma_\theta \geq 10°$), AoA+TDoA consistently reduced mean trajectory error by **25–45%**, maintaining sub-300 m accuracy even when AoA-only exceeded 500 m. A sweep over TDoA noise (10–100 ns, $\approx$3–30 m range) showed the fusion pipeline remained robust, highlighting tolerance to synchronization imperfections.

Beyond empirical results, a geometry-based dilution of precision (GDOP) analysis confirmed the information-theoretic advantage of fusion, with TDoA reducing ellipse eccentricity and improving conditioning under non-ideal sensor layouts. Comparative baselines with extended Kalman and particle filters demonstrated that while continuous filters achieve similar accuracy, the discrete beam-search approach offers competitive robustness at lower computational cost in multi-hypothesis settings.

Taken together, these findings reinforce the operational relevance of multi-modal RF fusion for electronic warfare and spectrum monitoring. By quantifying error reduction, robustness to noise, and geometric improvements, this study demonstrates that modest TDoA augmentation can materially extend the envelope of passive geolocation. Future work will focus on **real-world validation**, advanced channel models (e.g., multipath, fading), adaptive fusion strategies, and synchronization cost analysis to translate these gains into fielded systems.

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REFERENCES

[1] Y. Bar-Shalom, P. K. Willett, and X. Tian, *Tracking and Data Fusion: A Handbook of Algorithms*. YBS Publishing, 2011.

[2] S. Blackman and R. Popoli, *Design and Analysis of Modern Tracking Systems*. Artech House, 1999.

[3] X. Chen, K. Dogancay, and A. M. Bishop, "A survey on RF geolocation techniques," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 54, no. 5, pp. 2213–2245, 2018.

[4] W. H. Foy, "Position-location solutions by Taylor-series estimation," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-12, no. 2, pp. 187–194, 1976.

[5] S. Gezici, "A survey on wireless position estimation," *Wireless Personal Communications*, vol. 44, no. 3, pp. 263–282, 2008.

[6] K. C. Ho and Y. T. Chan, "Solution and performance analysis of geolocation by TDOA," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 29, no. 4, pp. 1311–1322, 1993.

[7] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," *IEEE Trans. Syst. Man Cybern. C*, vol. 37, no. 6, pp. 1067–1080, 2007.

[8] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero III, R. L. Moses, and N. S. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 54–69, 2005.

[9] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, 1986.

[10] D. J. Torrieri, "Statistical theory of passive location systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-20, no. 2, pp. 183–198, 1984.