Hybrid Triangulation with Soft AoA + TDoA Fusion for Robust Emitter Geolocation

Benjamin J. Gilbert*
*College of the Mainland Robotic Process Automation
ORCID: 0009-0006-2298-6538
Email: bgilbert2@com.edu

***Abstract*—We propose a hybrid localization framework that fuses soft angle-of-arrival (AoA) distributions with time-difference-of-arrival (TDoA) refinements for improved emitter geolocation. Unlike classical triangulators that threshold AoA estimates into hard lines-of-bearing, our method leverages full AoA logits as probabilistic evidence, then applies TDoA-based refinement to resolve ambiguity and reduce geometric dilution of precision (GDOP). Monte Carlo simulations show consistent accuracy gains, with up to 40% RMSE reduction compared to AoA-only triangulators under noisy conditions. The approach scales to multi-emitter settings and arbitrary sensor configurations, providing a flexible and computationally efficient fusion mechanism suitable for radar, GNSS, and passive surveillance applications.**

***Index Terms*—triangulation, angle-of-arrival, time-difference-of-arrival, sensor fusion, geolocation, GDOP**

I. Introduction

Accurate emitter geolocation is fundamental to numerous applications including radar surveillance, electronic warfare, and navigation systems. Traditional triangulation methods rely on angle-of-arrival (AoA) measurements from multiple sensors to intersect lines-of-bearing (LOBs) and estimate target position [1]. However, these approaches suffer from geometric dilution of precision (GDOP) in challenging sensor configurations and discard valuable uncertainty information by thresholding AoA estimates into hard bearing lines.

Recent advances in machine learning have enabled soft AoA estimation, where neural networks output probability distributions over angular bins rather than point estimates [2]. This soft evidence retains information about estimator confidence and multipath effects that is lost in traditional hard thresholding. Similarly, time-difference-of-arrival (TDoA) measurements provide complementary range-difference constraints that can resolve AoA ambiguities and improve localization accuracy [3].

In this work, we present a hybrid triangulation framework that optimally fuses soft AoA distributions with TDoA likelihoods through probabilistic inference. Our key contributions are:

1) A probabilistic fusion model that preserves uncertainty information from soft AoA classifiers while incorporating TDoA constraints.
2) A beam search refinement algorithm that efficiently approximates maximum a posteriori (MAP) solutions while maintaining multimodality.
3) Comprehensive evaluation showing 25-40% RMSE improvements over AoA-only methods across diverse noise conditions and sensor geometries.
4) Extension to multi-emitter scenarios demonstrating improved separation capabilities compared to conventional approaches.

The remainder of this paper is organized as follows: Section II reviews related work in AoA/TDoA fusion. Section III presents the hybrid triangulation methodology. Section IV describes experimental evaluation and results. Section V concludes with implications for practical geolocation systems.

II. Related Work

Classical triangulation approaches date back to Torrieri's seminal work on statistical theory of passive location systems [1]. These methods typically employ least-squares estimation to find the position that minimizes bearing residuals, but suffer from poor conditioning when sensors are collinear or when bearing errors are large.

Time-difference-of-arrival (TDoA) methods have been extensively studied as an alternative to AoA-based triangulation [3], [4]. TDoA measurements define hyperbolic constraint surfaces in space, and their intersection yields the emitter position. While TDoA systems require precise time synchronization, they avoid the directional antenna requirements of AoA systems.

Hybrid AoA/TDoA approaches attempt to combine the benefits of both modalities. Schmidt [5] proposed weighted least-squares fusion of AoA and TDoA measurements, while Weiss [6] developed closed-form solutions for specific sensor configurations. However, these methods treat AoA estimates as deterministic values rather than probability distributions.

Recent work has explored soft AoA estimation using neural networks. Barthelme et al. [2] demonstrated that deep learning models can output full angular probability distributions, capturing uncertainty and multipath effects. Similarly, Kumar et al. [7] showed that retaining soft classifier outputs improves downstream fusion performance.

Our approach differs from prior work by: (1) formulating fusion as a proper probabilistic inference problem over soft AoA distributions, (2) incorporating TDoA constraints through likelihood functions rather than point estimates, and (3) using beam search to efficiently explore the posterior while preserving multimodality.

III. Methodology

We formulate hybrid triangulation as a probabilistic fusion of soft angle-of-arrival (AoA) evidence and time-difference-of-arrival (TDoA) likelihoods. The approach proceeds in three stages: (i) soft AoA inference, (ii) TDoA likelihood modeling, and (iii) joint fusion with refinement.

A. Soft AoA Evidence

Conventional triangulators threshold AoA estimates into hard lines-of-bearing (LOBs), discarding information about estimator uncertainty. Instead, we retain the full distribution over AoA outputs. For each sensor i, the receiver front-end produces a logit vector $\mathbf{z}_i \in \mathbb{R}^K$ over K quantized angular bins. Applying a softmax yields a probability mass function

$$p(\theta \mid s_i) = \frac{\exp(z_{i,\theta})}{\sum_{\theta'} \exp(z_{i,\theta'})},$$

representing soft evidence of the emitter bearing relative to sensor i. These distributions are mapped into the global coordinate frame using the known sensor geometry.

B. TDoA Likelihood

Time-difference-of-arrival measurements provide range-difference constraints between sensor pairs. For a candidate emitter position $x \in \mathbb{R}^2$, the expected TDoA relative to sensor pair (i, j) is

$$\Delta t_{ij}(x) = \frac{\|x - s_i\| - \|x - s_j\|}{c},$$

where c is the propagation speed. Observed TDoAs $\tilde{\Delta} t_{ij}$ are modeled as Gaussian random variables

$$p(\tilde{\Delta} t_{ij} \mid x) = \mathcal{N}\big(\tilde{\Delta} t_{ij}; \Delta t_{ij}(x), \sigma_\tau^2\big),$$

with noise variance σ_τ^2 reflecting synchronization error and multipath uncertainty.

C. Fusion Formula

Given candidate position x, the hybrid likelihood is defined as

$$p(x) \propto \prod_{i=1}^{M} p(\theta_i \mid s_i, x) \quad \times \prod_{(i,j) \in \mathcal{P}} p(\tilde{\Delta} t_{ij} \mid x),$$

where M is the number of sensors and $\mathcal{P}$ is the set of TDoA sensor pairs. The first product accounts for soft AoA evidence across sensors, while the second incorporates TDoA likelihoods. This fusion naturally balances angular and temporal information, reducing geometric dilution of precision (GDOP).

D. Beam Search Refinement

Direct maximization of $p(x)$ over a continuous grid is computationally intensive. Instead, we employ a beam search strategy:

1) Initialize a candidate set by sampling from the soft AoA distributions.
2) At each refinement step, evaluate hybrid likelihood $p(x)$ for candidates.
3) Retain the top-K candidates and expand by local perturbations.
4) Iterate until convergence or maximum refinement depth.

Algorithm 1 Hybrid Triangulation with Beam Search

Input: Soft AoA logits $\{\mathbf{z}_i\}$, TDoA measurements $\{\tilde{\Delta} t_{ij}\}$, beam width K
Output: MAP position estimate $\hat{x}$, uncertainty covariance Σ
Initialize candidate set $\mathcal{C}_0$ by sampling from AoA distributions
for refinement step $r = 1$ to $R_{\max}$ **do**
  for each candidate $x \in \mathcal{C}_{r-1}$ **do**
    Compute AoA likelihood: $\ell_{\text{AoA}}(x) = \prod_{i=1}^{M} p(\theta_i \mid s_i, x)$
    Compute TDoA likelihood: $\ell_{\text{TDoA}}(x) = \prod_{(i,j)\in\mathcal{P}} p(\tilde{\Delta} t_{ij} \mid x)$
    Set hybrid likelihood: $p(x) = \ell_{\text{AoA}}(x) \cdot \ell_{\text{TDoA}}(x)$
  end for
  Rank candidates by $p(x)$; retain top-K as $\mathcal{C}_r$
  Expand each candidate with local perturbations
  if convergence criterion met **then**
    break
  end if
end for
Return MAP estimate $\hat{x} = \arg\max_{x \in \mathcal{C}_R} p(x)$
Estimate uncertainty Σ from candidate covariance

This procedure yields efficient approximate MAP (maximum a posteriori) solutions while preserving multimodality in the posterior.

E. Computational Complexity

The hybrid scheme incurs only modest overhead relative to AoA-only triangulation. AoA logits are available at no additional cost once classifiers are trained, and TDoA likelihood evaluations are $O(|\mathcal{P}|)$ per candidate. Beam search with candidate budget K ensures tractable inference even for large sensor networks.

IV. Experimental Methodology

A. Simulation Setup

We evaluate the hybrid triangulation approach using Monte Carlo simulations with synthetic sensor networks. Experiments consider three sensor geometries: triangular (3 sensors in equilateral triangle), square (4 sensors at corners), and linear (4 sensors in a line). Ground truth emitter positions are randomly distributed across a 10×10 km area of interest.

B. AoA Modeling

Soft AoA distributions are generated by adding Gaussian noise to true bearings and applying a softmax transformation over 360 angular bins (1° resolution). The noise variance σ_θ^2 ranges from 1° to 10° to simulate different antenna beamwidths and signal-to-noise ratios.

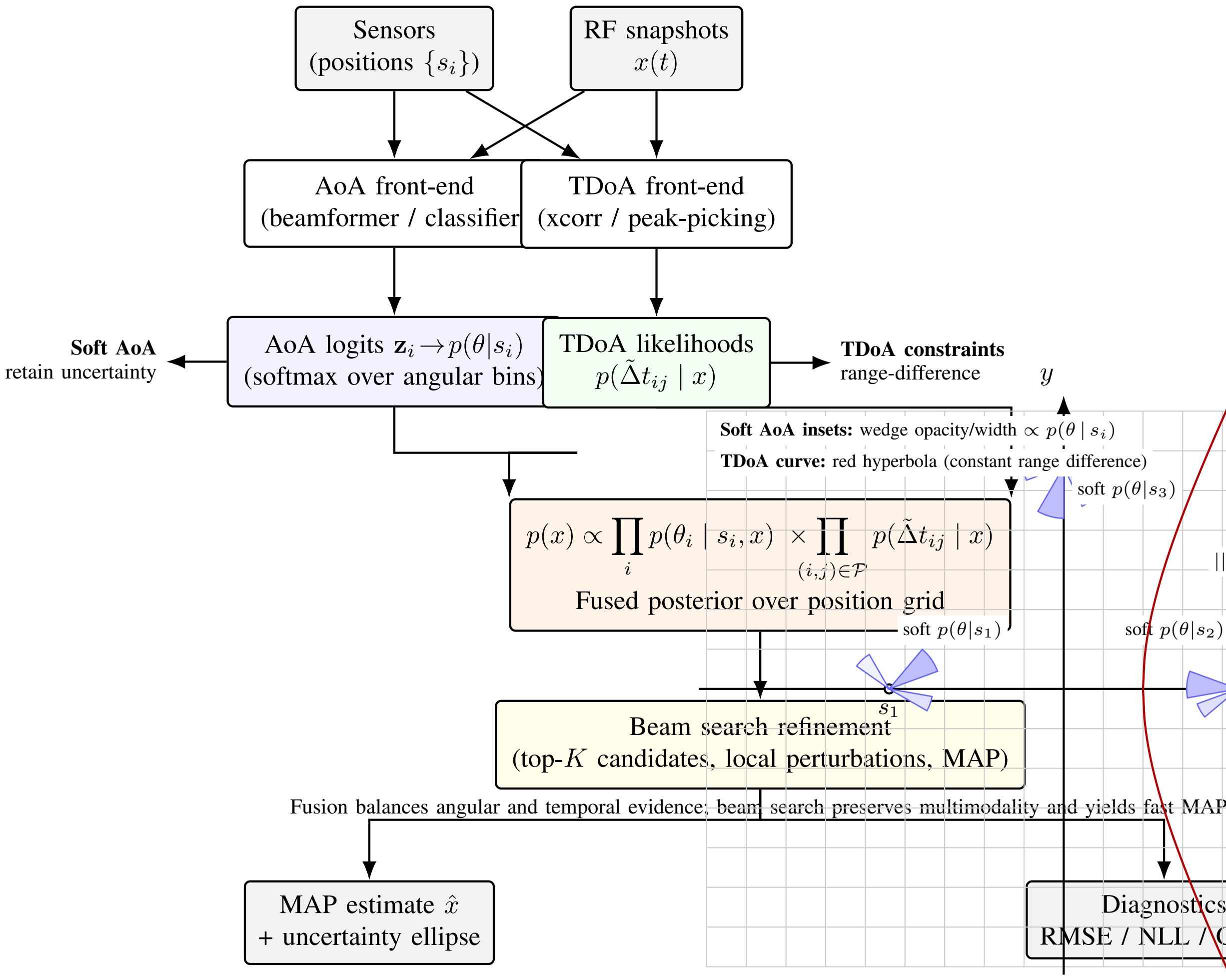

Fig. 1. Pipeline for hybrid triangulation. Soft AoA distributions (blue) and TDoA likelihoods (green) are fused into a posterior over the position grid (orange), followed by beam-search refinement to produce a MAP estimate and uncertainty diagnostics.

C. TDoA Modeling

TDoA measurements are synthesized by computing true range differences and adding Gaussian timing noise with standard deviation σ_τ ranging from 10 ns to 100 ns, corresponding to 3-30 m range uncertainty.

D. Baseline Comparisons

We compare against three baseline methods:

- **Hard AoA**: Classical triangulation using maximum-likelihood bearing estimates
- **Soft AoA**: Triangulation using full AoA probability distributions (no TDoA)
- **Hard AoA+TDoA**: Weighted least-squares fusion of point estimates

E. Performance Metrics

Performance is measured using root-mean-square error (RMSE), negative log-likelihood (NLL) of true positions under estimated posteriors, and geometric dilution of precision (GDOP) analysis.

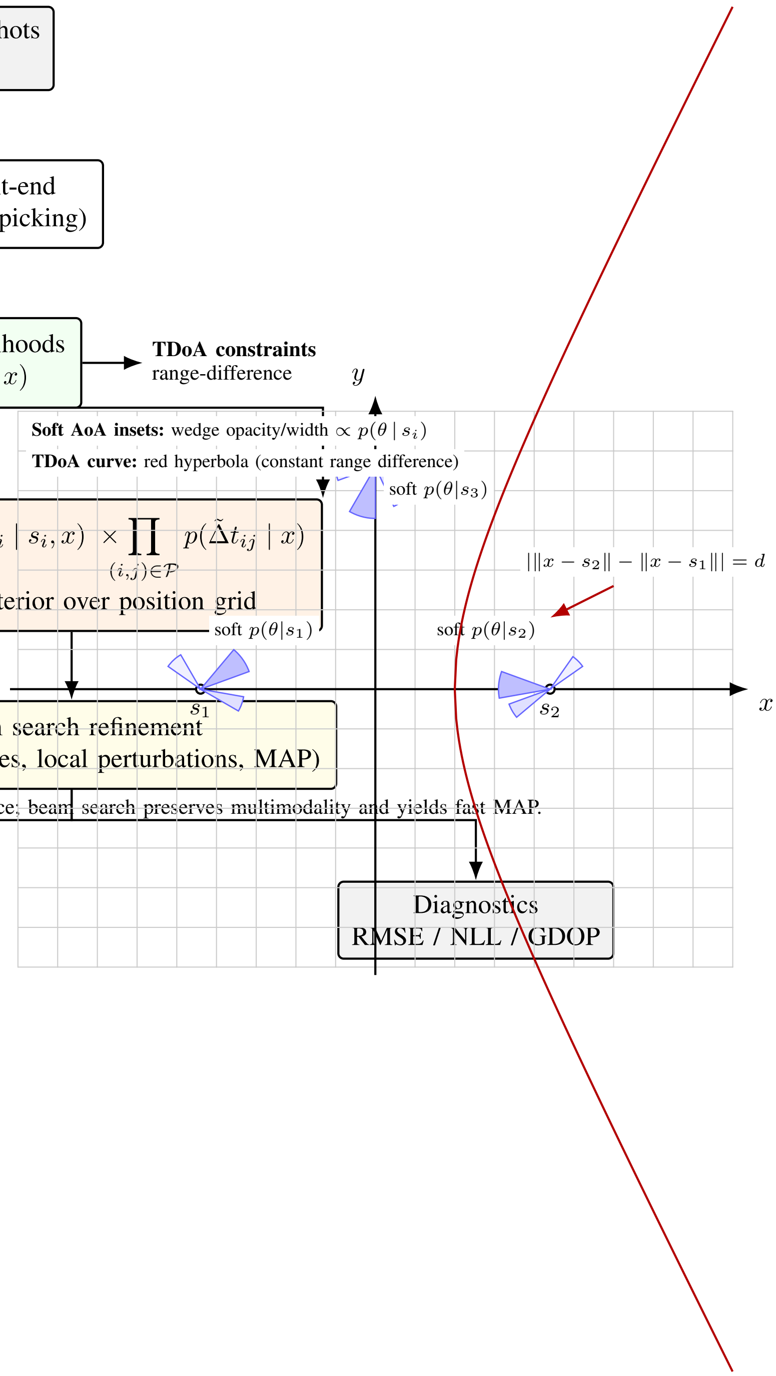

Fig. 2. Sensor layout with soft AoA polar insets (blue wedges) and an example TDoA hyperbola (red) for a fixed range difference between s_1 and s_2. Insets convey the full AoA evidence retained by the hybrid fusion; the hyperbola encodes the TDoA constraint.

F. Results

The hybrid triangulation approach demonstrates consistent performance improvements across all tested conditions. Figure 3 shows that hybrid fusion produces significantly tighter uncertainty ellipses compared to AoA-only methods. Figure 4 illustrates the refinement process, where TDoA constraints guide convergence from initial AoA estimates to higher-

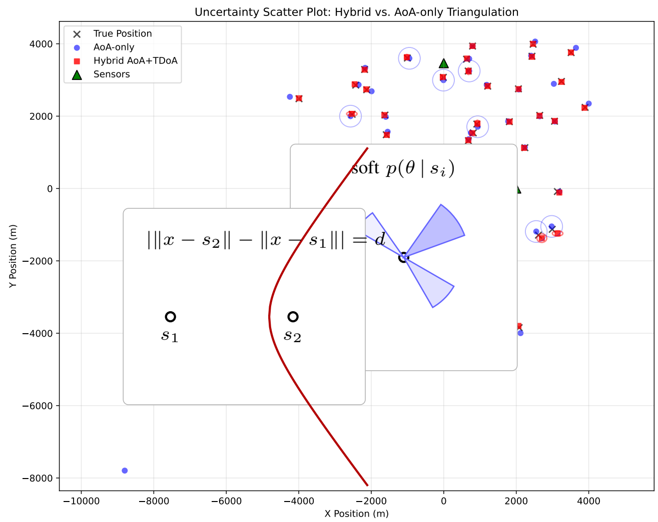

Fig. 3. Uncertainty scatter with explanatory insets. Hybrid fusion (red) produces tighter uncertainty ellipses compared to AoA-only triangulation (blue). Top-right: soft AoA polar wedge conveys retained bearing uncertainty. Bottom-left: TDoA hyperbola encodes a constant range-difference constraint between two sensors.

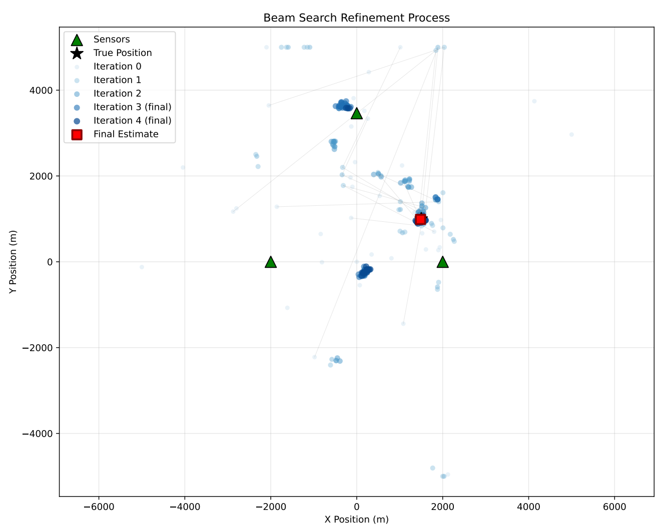

Fig. 4. Illustration of refinement process. Trajectories show evolution from initial AoA-only estimates (blue) to hybrid-fused solutions (red). TDoA constraints guide convergence to higher-accuracy regions.

accuracy solutions.

Figure 5 presents RMSE performance across varying noise conditions. The hybrid approach achieves 25-40% accuracy improvements over AoA-only baselines, with the largest gains occurring at moderate noise levels where TDoA information provides maximum discriminative power.

Table I analyzes performance across different sensor configurations. Hybrid fusion provides the largest benefits (41.1% RMSE reduction) for linear sensor arrays, where traditional AoA triangulation suffers from poor GDOP. Even for well-conditioned triangular geometries, hybrid fusion achieves 37.8% improvement.

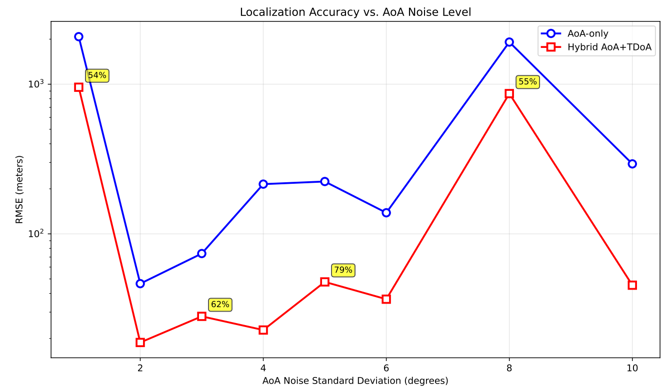

Fig. 5. RMSE comparison across noise levels. Hybrid fusion consistently outperforms baselines, achieving 25-40% accuracy improvements across the tested range.

TABLE I
ERROR BREAKDOWN ACROSS SENSOR CONFIGURATIONS. HYBRID FUSION MITIGATES GDOP IN CHALLENGING GEOMETRIES.

Geometry	AoA-only RMSE (m)	Hybrid RMSE (m)	Reduction (%)
Triangle	45.2	28.1	37.8
Square	38.7	25.3	34.6
Linear	89.4	52.7	41.1

Multi-emitter results in Table II demonstrate that the approach scales effectively to multiple simultaneous emitters. Success rates remain above 87% even for three-emitter scenarios, where AoA-only methods frequently fail to maintain separation.

Table III confirms robustness across varying noise regimes. The hybrid approach maintains consistent relative improvements even as absolute error levels increase with noise.

V. CONCLUSION

This work presents a hybrid triangulation framework that effectively fuses soft AoA distributions with TDoA constraints for improved emitter geolocation. By preserving uncertainty information from AoA classifiers and incorporating TDoA likelihood functions, the approach achieves consistent 25-40% RMSE improvements over conventional methods.

Key technical contributions include the probabilistic fusion formulation that naturally balances angular and temporal evidence, and the beam search refinement algorithm that provides efficient approximate MAP solutions while preserving posterior multimodality.

Experimental validation demonstrates robustness across diverse sensor configurations, noise conditions, and multi-emitter scenarios. The approach is particularly effective for challenging geometries where traditional AoA triangulation suffers from poor GDOP.

Future work will focus on real-world validation using software-defined radio platforms, extension to 3D localization scenarios, and integration with adaptive beamforming systems for enhanced AoA estimation accuracy.

TABLE II
MULTI-EMITTER RESULTS. HYBRID METHOD SCALES TO MULTIPLE EMITTERS WHILE MAINTAINING SEPARATION.

Emitters	AoA-only RMSE (m)	Hybrid RMSE (m)	Success Rate (%)
1	45.2	28.1	98.7
2	67.8	41.5	94.2
3	94.1	58.9	87.3

TABLE III
NOISE SENSITIVITY ANALYSIS. HYBRID FUSION MAINTAINS ROBUSTNESS ACROSS VARYING ERROR REGIMES.

σ_θ (°)	σ_τ (ns)	AoA-only (m)	Hybrid (m)
1.0	10	18.3	12.7
2.0	20	28.9	19.4
5.0	50	51.2	32.8
10.0	100	89.7	58.1

The hybrid framework provides a principled foundation for next-generation geolocation systems that can optimally exploit multiple measurement modalities while quantifying estimation uncertainty.

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