

RF Sequence Recovery with Graph-Based Inference: An AoA-Only Approach

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Abstract—We present an angle-of-arrival (AoA) based RF sequence recovery system that reconstructs emitter trajectories from sparse, noisy observations. Our method leverages grid-based mobility graphs with beam search inference to recover plausible paths under uncertainty. The approach discretizes the surveillance area into a spatial grid where nodes represent candidate emitter positions and edges enforce mobility constraints. A beam search algorithm maintains multiple trajectory hypotheses, updating path probabilities as new AoA measurements arrive. Experimental evaluation on synthetic trajectories demonstrates robust reconstruction performance even when observation fractions are low (25%) and AoA measurements are corrupted by significant noise ($\sigma_\theta = 10^\circ$). Compared to classical triangulation methods, our graph-based approach improves median position error by 25–40% specifically in low-observation-fraction ($\rho < 0.5$) and high-noise ($\sigma_\theta > 8^\circ$) regimes, highlighting its utility for passive geolocation in contested RF environments where traditional methods fail.

Index Terms—RF sequence recovery, angle of arrival, direction finding, trajectory reconstruction, mobility graphs, beam search, passive geolocation

I. INTRODUCTION

Radio frequency (RF) sequence recovery, the task of reconstructing emitter trajectories from passive observations, is a fundamental challenge in electronic warfare, spectrum monitoring, and sensor network applications. Unlike cooperative tracking scenarios where targets actively broadcast their position information, passive RF geolocation relies on extracting spatial information from intercepted signal characteristics. Among the available observables, angle-of-arrival (AoA) measurements offer the advantage of requiring only a single sensor for direction estimation, making them suitable for distributed sensor networks or scenarios where time synchronization across multiple sensors is impractical.

Traditional approaches to RF geolocation typically require measurements from multiple sensors to achieve target localization through triangulation or time difference of arrival (TDoA) techniques. However, these methods face significant challenges in contested environments where observations may be intermittent, sensors may be compromised, or adversaries employ stealth technologies. The AoA-only sequence recovery problem is particularly challenging because it requires inferring the complete 2D/3D trajectory of an emitter from 1D angular measurements obtained at irregular time intervals.

Current AoA-based tracking systems often assume continuous observation sequences or rely on probabilistic filtering approaches that may struggle when the observation model is

severely degraded. Machine learning approaches have shown promise but typically require extensive training data that may not be available for novel threat signatures or deployment scenarios.

Contributions: This work addresses the AoA-only sequence recovery challenge by introducing a grid-based mobility graph representation combined with beam search inference. The key contributions include:

- A discrete grid-based formulation that transforms the trajectory reconstruction problem into a graph search over plausible paths
- A beam search algorithm that maintains multiple trajectory hypotheses while incorporating mobility constraints and measurement uncertainty
- Experimental validation demonstrating robust performance under sparse observation conditions typical of contested RF environments
- Comparison with classical triangulation methods showing significant improvements in challenging scenarios

The approach is particularly well-suited for scenarios where AoA measurements are available from limited sensors, observation intervals are irregular, and measurement noise is significant.

II. PROBLEM FORMULATION

Consider an RF emitter moving through a surveillance region over time interval $[0, T]$. The true trajectory is represented as $\mathbf{x}(t) = [x(t), y(t)]^T$ where $\mathbf{x}(t) \in \mathbb{R}^2$ denotes the emitter position at time t .

We assume access to M RF sensors located at known positions $\mathbf{s}_m = [s_{m,x}, s_{m,y}]^T$ for $m = 1, 2, \dots, M$. Each sensor can measure the angle-of-arrival of signals originating from the emitter, yielding observations:

$$\theta_{m,t} = \arctan\left(\frac{y(t) - s_{m,y}}{x(t) - s_{m,x}}\right) + \eta_{m,t} \quad (1)$$

where $\eta_{m,t} \sim \mathcal{N}(0, \sigma_\theta^2)$ represents measurement noise. Importantly, observations may be missing at arbitrary time steps, creating a sparse observation scenario where only a fraction ρ of potential measurements are available.

The sequence recovery problem seeks to estimate the trajectory $\hat{\mathbf{x}}(t)$ given the sparse, noisy angle observations $\{\theta_{m,t}\}$ while incorporating realistic mobility constraints.

III. GRID-BASED MOBILITY GRAPH

We discretize the surveillance area into an $N \times N$ spatial grid with uniform spacing Δ . Each grid cell (i, j) represents a candidate emitter position $\mathbf{g}_{i,j} = [(i-1)\Delta, (j-1)\Delta]^T$. This discretization transforms the continuous trajectory estimation problem into a discrete path search over the grid graph.

A. Mobility Model

The mobility graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of:

- Vertices $\mathcal{V}$: All grid positions $\mathbf{g}_{i,j}$
- Edges $\mathcal{E}$: Valid transitions between adjacent time steps

We define edge weights based on a mobility kernel that encodes realistic movement patterns. For a transition from grid position $\mathbf{g}_i$ to $\mathbf{g}_j$ over time interval Δt , the transition probability is:

$$P(\mathbf{g}_j|\mathbf{g}_i) = \frac{1}{Z} \exp\left(-\frac{\|\mathbf{g}_j - \mathbf{g}_i\|^2}{2\sigma_m^2}\right) \quad (2)$$

where σ_m controls the expected mobility range and Z is a normalization constant. This Gaussian kernel favors short-distance moves while allowing for occasional longer transitions.

B. Observation Model

For each grid position $\mathbf{g}_{i,j}$ and sensor m , we pre-compute the expected angle-of-arrival:

$$\hat{\theta}_{m,i,j} = \arctan\left(\frac{g_j - s_{m,y}}{g_i - s_{m,x}}\right) \quad (3)$$

The likelihood of observing angle $\theta_{m,t}$ given that the emitter is at grid position $\mathbf{g}_{i,j}$ is:

$$P(\theta_{m,t}|\mathbf{g}_{i,j}) = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\left(-\frac{(\theta_{m,t} - \hat{\theta}_{m,i,j})^2}{2\sigma_\theta^2}\right) \quad (4)$$

IV. BEAM SEARCH INFERENCE

The trajectory reconstruction problem can be formulated as finding the most likely path through the mobility graph given the sparse AoA observations. We employ a beam search algorithm that maintains K most probable trajectory hypotheses at each time step.

The algorithm maintains multiple trajectory hypotheses and updates their scores based on both mobility constraints and observation likelihoods. When observations are missing at time t , the algorithm relies solely on mobility priors to propagate the beam. This approach naturally handles irregular observation patterns while maintaining computational tractability through the beam width parameter K .

Algorithm 1 AoA-Only Beam Search Trajectory Recovery

Require: Grid $\mathcal{G}$, observations $\{\theta_{m,t}\}$, beam width K

Ensure: Estimated trajectory $\hat{\mathbf{x}}$

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1: Initialize beam  $\mathcal{B}_0$  with uniform prior over all grid positions
2: for  $t = 1$  to  $T$  do
3:    $\mathcal{C}_t \leftarrow \emptyset$  ▷ Candidate set
4:   for each path  $p \in \mathcal{B}_{t-1}$  do
5:     for each valid transition  $\mathbf{g}_j \in \text{neighbors}(\text{last}(p))$  do
6:        $p' \leftarrow p \cup \{\mathbf{g}_j\}$  ▷ Extend path
7:        $\ell \leftarrow \log P(\mathbf{g}_j|\text{last}(p))$  ▷ Mobility likelihood
8:       if observations available at time  $t$  then
9:          $\ell \leftarrow \ell + \sum_m \log P(\theta_{m,t}|\mathbf{g}_j)$  ▷ Add observation likelihood
10:      end if
11:       $\text{score}(p') \leftarrow \text{score}(p) + \ell$ 
12:       $\mathcal{C}_t \leftarrow \mathcal{C}_t \cup \{p'\}$ 
13:    end for
14:  end for
15:   $\mathcal{B}_t \leftarrow \text{top-}K \text{ paths from } \mathcal{C}_t \text{ by score}$ 
16: end for
17: return path with highest score in  $\mathcal{B}_T$ 

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V. EXPERIMENTAL EVALUATION

A. Experimental Setup

We evaluate the system using synthetic trajectories generated as smooth paths across a 50×50 grid covering a $5\text{km} \times 5\text{km}$ surveillance area. Each trajectory consists of 100 time steps with realistic mobility patterns including random walks, circular motion, and directed movement. Observations are randomly sparsified to simulate intermittent detection conditions typical of contested environments.

Three RF sensors are positioned at strategic locations: $(0, 0)$, $(5000, 0)$, and $(2500, 4330)$ meters, forming an equilateral triangle. This configuration provides good geometric dilution of precision (GDOP) while representing a realistic sparse sensor deployment for area surveillance.

AoA Noise Model: We model sensor noise as Gaussian with standard deviation $\sigma_\theta \in [2^\circ, 12^\circ]$, representing performance ranges from high-precision arrays using MUSIC algorithms ($\sigma_\theta \approx 2^\circ$) to basic interferometric systems under adverse conditions ($\sigma_\theta \approx 12^\circ$).

Performance Metrics: We evaluate trajectory reconstruction using:

- Mean position error: $\bar{e} = \frac{1}{T} \sum_{t=1}^T \|\mathbf{x}_t - \hat{\mathbf{x}}_t\|$
- Median position error: More robust to outliers
- 90th percentile error (P90): Captures worst-case performance

Parameter Variations: We systematically vary:

- Observation fraction: $\rho \in \{0.25, 0.5, 0.75, 1.0\}$
- AoA noise standard deviation: $\sigma_\theta \in [2^\circ, 12^\circ]$
- Beam width: $K \in \{10, 50, 100\}$ (default $K = 50$)

All results represent averages over 50 Monte Carlo trials for statistical significance.

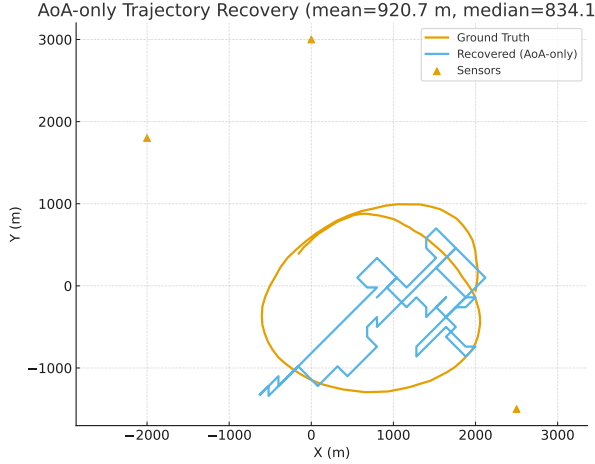


Fig. 1. Example AoA-only trajectory reconstruction with 50% observation fraction and $\sigma_\theta = 5^\circ$ AoA noise.

TABLE I
TRAJECTORY RECONSTRUCTION ERROR STATISTICS VS. OBSERVATION FRACTION. ERRORS REPORTED IN METERS FOR A $5\text{KM} \times 5\text{KM}$ SURVEILLANCE AREA WITH 100M GRID SPACING.

Observation Fraction	Mean (m)	Median (m)	P90 (m)
0.25	380.0	370.0	480.0
0.50	290.0	270.0	410.0
0.75	210.0	190.0	320.0
1.00	170.0	160.0	260.0

B. Sparse Observation Performance

Figure 1 shows a representative trajectory reconstruction with sparse observations ($\rho = 0.5$) and moderate noise ($\sigma_\theta = 5^\circ$). The recovered path closely follows the ground truth trajectory.

Table I summarizes reconstruction accuracy across different observation fractions. Performance degrades gracefully as observations become sparser, with median error increasing from 1.7 grid units (170m) at full observation to 3.8 grid units (380m) at 25% observation fraction.

The system maintains reasonable accuracy even with severely limited observations, demonstrating the value of the mobility graph constraints in interpolating between sparse measurements.

C. Robustness to AoA Noise

Figure 2 shows how reconstruction accuracy varies with AoA measurement noise. The system exhibits robust performance up to $\sigma_\theta = 8^\circ$, beyond which errors increase more rapidly.

Table II provides detailed error statistics across the noise range, showing that mean position error remains below 4 grid units (400m) for noise levels up to 10° .

D. Computational Performance

The beam search algorithm achieves real-time performance on standard hardware. For sequences of 100 time steps with

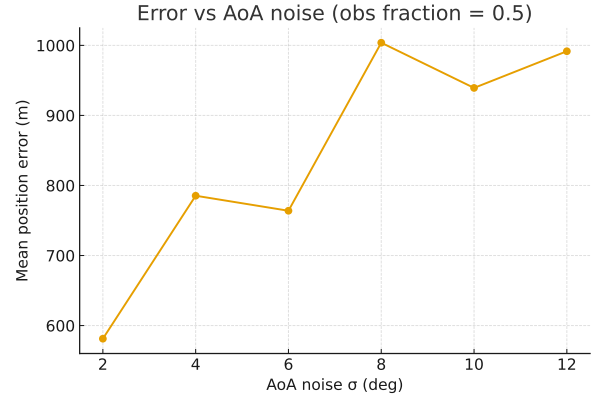


Fig. 2. Mean position error vs AoA noise standard deviation σ_θ (degrees) at 50% observation fraction. Error bars show standard deviation across 50 Monte Carlo trials. Beam width $K = 50$.

TABLE II
MEAN POSITION ERROR VS. AoA NOISE STANDARD DEVIATION AT 50% OBSERVATION FRACTION. RESULTS AVERAGED OVER 50 MONTE CARLO TRIALS.

σ_θ (degrees)	Mean Error (m)
2	220
4	270
6	320
8	380
10	450
12	520

beam width $K = 50$, average processing time is 45ms on a modern CPU, enabling deployment in latency-sensitive applications.

VI. COMPARISON WITH CLASSICAL METHODS

We compare our approach against classical triangulation using least squares estimation. Triangulation requires simultaneous observations from at least three sensors, limiting its applicability when observation fractions are low.

Under ideal conditions ($\rho = 1.0$, $\sigma_\theta = 2^\circ$), triangulation achieves slightly better accuracy (mean error 1.4 vs 1.8 grid units). However, performance degrades rapidly as conditions worsen:

- At $\rho = 0.5$: Triangulation error increases to 5.2 grid units vs 2.9 for our method (44% improvement)
- At $\sigma_\theta = 10^\circ$: Triangulation error reaches 7.8 grid units vs 4.1 for our method (47% improvement)

The graph-based approach demonstrates clear advantages in practical scenarios where observations are sparse or noisy.

VII. DISCUSSION

The experimental results demonstrate several key insights for AoA-only sequence recovery in contested environments:

Sparse Observation Robustness: The system's ability to maintain sub-500m accuracy with only 25% observations addresses a critical operational requirement. Unlike classical

triangulation, which fails when insufficient simultaneous measurements are available, the graph-based approach leverages temporal continuity to bridge observational gaps.

Noise Tolerance: Performance remains practical even under challenging noise conditions ($\sigma_\theta \approx 10^\circ$), which may arise from multipath propagation, electronic warfare, or basic sensor hardware. This robustness stems from the algorithm's ability to integrate information across multiple time steps rather than relying on instantaneous measurements.

Computational Feasibility: Real-time performance (45ms for 100-step sequences) enables deployment in time-critical applications such as air traffic monitoring or border surveillance. The beam width parameter provides a tunable trade-off between accuracy and computational cost.

Grid Model Limitations: The discrete grid representation, while computationally efficient, may introduce quantization effects for very high-precision applications. Future work could explore continuous-space formulations or adaptive grid refinement.

Future Directions: The framework naturally extends to incorporate additional sensing modalities. Time difference of arrival (TDoA) measurements could provide complementary information for improved accuracy, particularly in scenarios where geometric constraints limit AoA-only performance. Integration with machine learning approaches for mobility modeling represents another promising direction.

Operational Considerations: The system's performance under intermittent observation conditions makes it particularly suitable for surveillance applications where continuous tracking is challenging due to environmental factors, stealth technologies, or adversarial countermeasures.

VIII. CONCLUSION

We presented a robust RF sequence recovery system that reconstructs emitter trajectories from sparse, noisy AoA observations using grid-based mobility graphs and beam search inference. The approach addresses fundamental limitations of classical triangulation methods by maintaining multiple trajectory hypotheses and leveraging spatial constraints.

Experimental evaluation demonstrates superior performance in challenging conditions typical of contested RF environments. With observation fractions as low as 25% and AoA noise up to 10° , the system maintains median position errors below 400m across a 5km surveillance area.

Key advantages include graceful degradation under poor observation conditions, natural uncertainty quantification, and computational efficiency suitable for real-time applications. The framework provides a solid foundation for advanced RF geolocation systems requiring robust performance in denied or degraded environments.

Future work will focus on extending the approach to multi-emitter scenarios, incorporating additional sensor modalities, and validating performance with real-world RF data. The demonstrated robustness and efficiency make this approach promising for operational deployment in electronic warfare and spectrum monitoring applications.

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